

# Enhanced Robustness of Generic Model Control Using Derivative Feedback

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With the inherent nonlinear nature of chemical processes, the utility of a control structure which incorporates fundamental process models becomes evident. Implementation of linearized model based control may cause a lack of robustness or even instability within the operating range of these processes.

A control structure which allows fundamental process models to be imbedded within the controller is Generic Model Control (GMC) (Lee and Sullivan, 1988). This controller has been shown to yield robust control for a wide range of process nonlinearity and a degree of process model mismatch (Riggs and Rhinehart, 1988). Recent efforts to apply GMC to an industrial process (Bezanon et al., 1989) have shown that GMC lacks robustness for critical and over-damped closed-loop specifications. In this paper, model error compensation using derivative feedback is used to enhance robustness. The proposed structure is termed RGMC (Robust Generic Model Control) due to its similarity in model error compensation to Internal Model Control (IMC) (Garcia and Morari, 1982; Morari and Zafiriou, 1989), Dynamic Matrix Control (DMC) (Cutler and Ramaker, 1980), and Model Algorithmic Control (MAC) (Rouhani and Mehra, 1982). These control structures estimate and compensate for the error between the process and the process model output. The RGMC structure is distinct in that RGMC estimates and compensates for the error between the process and process model output time derivative.

The organization of this paper is as follows; an analysis of GMC control theory and its limitations are presented, the control structure for RGMC is introduced and its merits discussed, and robustness of RGMC is demonstrated and compared to GMC on two examples.

## Generic Model Control

For ease of presentation, the single input/single output case will be presented; however, all concepts presented can be extended to the multivariable case.

Generic Model Control is an optimal control approach to forcing the process output rate to match a reference rate. Based on physical laws, a set of first-order equations can be written to model the system:

$$\dot{x} = f(x, u, t)$$

$$y_m = g(x)$$

(The notation  $y_m$  is used to denote the output of the process model.)

The reference rate for the process is generated from the set point deviation as,

$$r^* = k_1(y^* - y) + k_2 \int (y^* - y) dt \quad (1)$$

where  $r^*$  is the desired process output "rate of change,"  $y^*$  is the setpoint, and  $k_1, k_2$  are the generic control loop constants. Note that the reference rate is proportional to the distance from the setpoint, and includes integral action to eliminate offset and increase robustness (Lee and Sullivan, 1988). The control signal is obtained by setting

$$r^* = \dot{y}_m = g(\dot{x}) \quad (2)$$

and solving for the input,  $u$ . The block diagram for the GMC structure is shown in Figure 1a.

A major advantage of GMC is that a generalized or "generic" response profile may be specified through the choice of  $k_1$  and  $k_2$ . The profile obtained is independent of the particular process, provided an exact inverse model for the process is available. The analysis which follows demonstrates the effect of modeling error on the closed loop.

The closed-loop system equation is obtained by combining Eq. 1 and 2.

$$\dot{y}_m = k_1(y^* - y) + k_2 \int (y^* - y) dt \quad (3)$$

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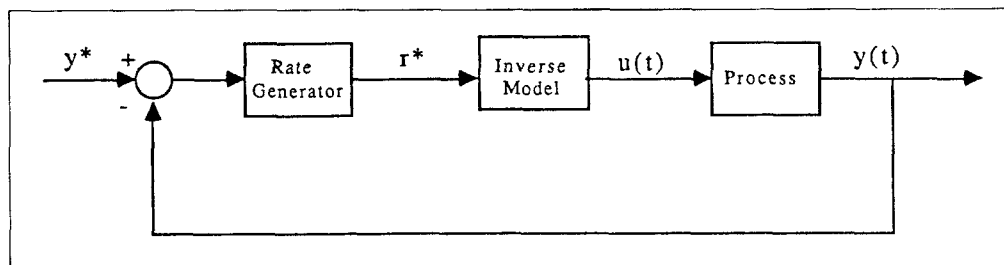


Figure 1a. GMC block diagram.

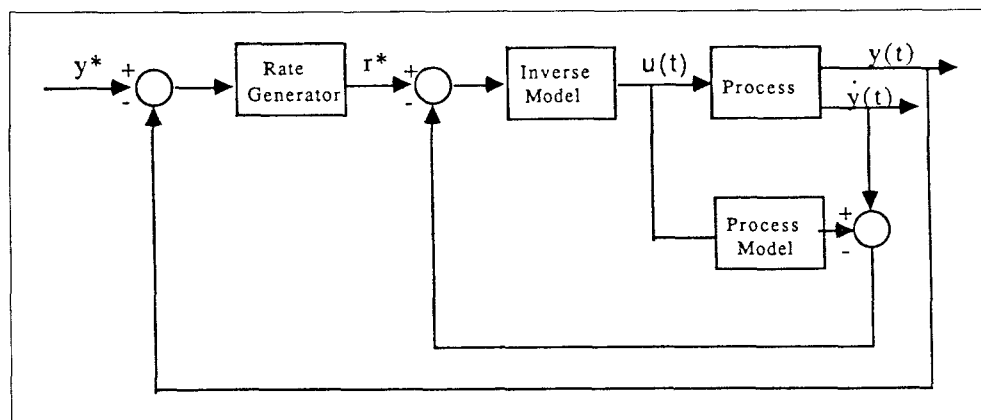


Figure 1b. RGMC block diagram.

Let  $e(t)$  represent modeling errors, due to model mismatch or unmeasured disturbances. The model output rate is related to the process as,

$$\dot{y}_m(t) = \dot{y}(t) + e(t) \quad (4)$$

Combining Eqs. 3 and 4, taking the Laplace transforms and rearranging results in the transfer function,

$$Y(s) = \frac{k_1 s + k_2}{s^2 + k_1 s + k_2} Y^*(s) - \frac{s}{s^2 + k_1 s + k_2} E(s) \quad (5)$$

Assuming a perfectly modeled system,  $e(t) = 0$ , a second-order response to setpoint changes is obtained. The system is not the classical second-order process (Stephanopoulos, 1984), but the controller constants can be related to frequency and damping ratio characteristics. A tuning map can easily be constructed and used to pick control constants which result in the desired shape and speed of response (Lee and Sullivan, 1988).

Any errors in modeling the process are intended to be compensated for by the integral action of the controller. However for an "overdamped" specification,  $k_2$  is negligible compared to  $k_1$ . As a result, the integral action is reduced to the point where offset elimination is extremely sluggish (this will be shown in Example 1). Consider the case of  $k_2 = 0$ . For a stepwise error for  $e(t)$ , application of the final value theorem to Eq. 5 predicts a steady-state offset of  $|e|/k_1$ .

### Robust Generic Model Control

The Robust Generic Model Control structure is proposed,

$$\dot{y}_m(t) = r^* - [\dot{y}(t) - \dot{y}_m(t - \Theta)] \quad (6)$$

where  $r^*$  was defined by Eq. 1 and  $\Theta$  is an arbitrary but short length of time, typically the sample time. In the form given by Eq. 6, RGMC is represented as GMC with modeling error compensation. The modeling error is estimated and used as feedback in an internal loop, as shown by the block diagram of this structure in Figure 1b.

Making the substitution of Eq. 4,

$$\dot{y}_m(t) = \dot{y}(t) + e(t),$$

taking the Laplace transforms and rearranging results in the transfer function,

$$Y(s) = \frac{k_1 s + k_2}{s^2 + k_1 s + k_2 + (1 - e^{-\Theta s})s^2} Y^*(s) - \frac{(1 - e^{-\Theta s})s}{s^2 + k_1 s + k_2 + (1 - e^{-\Theta s})s^2} E(s) \quad (10)$$

Application of the final value theorem shows that the process output asymptotically approaches the setpoint and no offset occurs for any value of  $k_2$ . Also note that as  $\Theta$  is decreased, the dependence of the response on modeling errors is decreased.

For the case of perfect modeling, RGMC results in the transfer function,

$$\frac{Y(s)}{Y^*(s)} = \frac{k_1 s + k_2}{s^2 + k_1 s + k_2 + (1 - e^{-\Theta s})s^2} \quad (11)$$

which demonstrates how the choice of  $\Theta$  alters the specified response of RGMC compared to GMC for the  $k_1$ ,  $k_2$  values used. However, for a small  $\Theta$ , the alteration in response is minimal and the use of the GMC tuning map is appropriate.

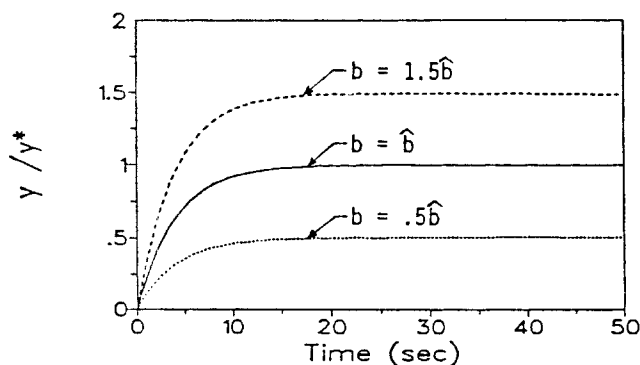


Figure 2a. GMC response for Example 1.

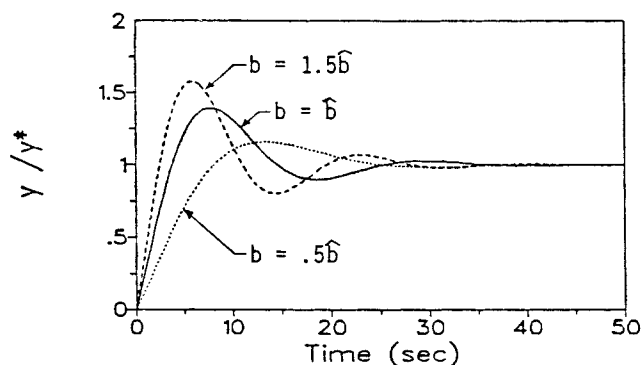


Figure 3a. GMC response for Example 2.

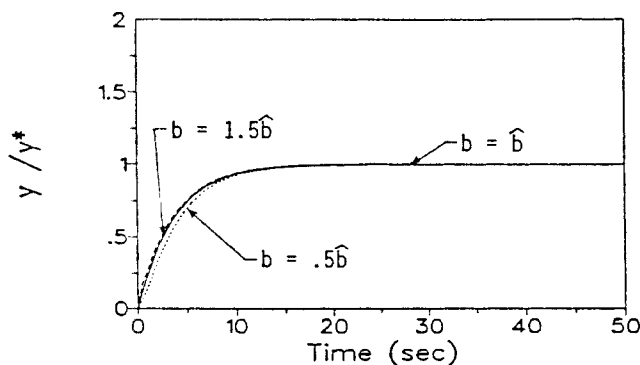


Figure 2b. RGMC response for Example 1.

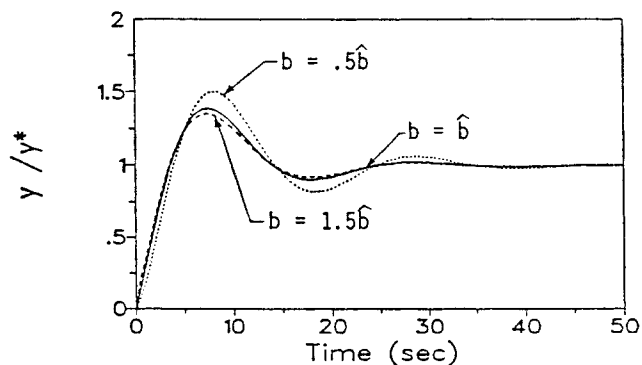


Figure 3b. RGMC response for Example 2.

### Example 1

Consider a linear process,

$$\dot{y} = ay + bu$$

that has been modeled as,

$$\dot{y}_m = \hat{a}y + \hat{b}u$$

with  $\hat{a} = -0.25$  and  $\hat{b} = 0.50$ . Suppose that  $a$  is constant and estimated correctly, but  $b$  is not constant, varying by as much as 50% from the estimated value. Three values for  $b$  ( $.5\hat{b}$ ,  $\hat{b}$ , and  $1.5\hat{b}$ ) were used in the process simulation. An overdamped closed-loop response is specified by using the generic control loop parameters of  $k_1 = 0.25 \text{ s}^{-1}$ ,  $k_2 = 0.0001 \text{ s}^{-2}$ . The controller was applied with a sample time of 0.5 s. The on-line estimate of  $y$  was obtained by an Euler approximation.

For a step change in setpoint, the GMC control response is shown in Figure 2a. The RGMC response is shown in Figure 2b. Because of the small  $k_2$  value, the GMC response for the two cases with modeling errors produced a steady-state offset; the offset elimination due to integral action is so extremely sluggish as to be nonexistent. The RGMC response conforms closely to the specified response for the variations in the process, demonstrating the robust character of the RGMC structure.

### Example 2

Consider the same process and model used in Example 1. For an underdamped closed-loop response, the control parameters were set as  $k_1 = 0.25 \text{ s}^{-1}$ ,  $k_2 = 0.10 \text{ s}^{-2}$ .

The response of the system under GMC and RGMC is shown in Figures 3a and 3b. By increasing the value of  $k_2$ , an oscillatory response is specified and the integral action produces the offset elimination intended by GMC. Although both GMC and RGMC produce offset elimination for the variations in the process, RGMC displays improved robustness by more closely matching the specified response.

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### Notation

- $a, b$  = process or system parameters
- $\hat{a}, \hat{b}$  = process model parameters
- $e(t)$  = process model error
- $E(s)$  = Laplace transform of  $e(t)$
- $f, g$  = denote arbitrary functions
- $k_1, k_2$  = generic control loop constants
- $r^*$  = reference rate signal
- $s$  = Laplace transform variable
- $t$  = time variable
- $u$  = process input
- $x$  = system state variable
- $y$  = process or system output
- $Y(s)$  = Laplace transform of the process output
- $y_m$  = process model output
- $y^*$  = process setpoint
- $Y^*(s)$  = Laplace transform of the setpoint
- $\dot{y}$  = process or system output rate
- $\dot{y}_m$  = process model output rate

## Greek letters

$\Theta$  = time interval

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